

NON-LINEAR FINITE ELEMENT ANALYSIS OF ELASTOMERS

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MATERIAL TESTING AND CHARACTERIZATION

1.1 Introduction

The application of computational mechanics analysis techniques to elastomers presents unique challenges in modeling the following characteristics:

- The load-deflection behavior of an elastomer is markedly non-linear.
- The recoverable strains can be as high 400 % making it imperative to use the large deflection theory.
- The stress-strain characteristics are highly dependent on temperature and rate effects are pronounced.
- Elastomers are nearly incompressible.
- Viscoelastic effects are significant.

The ability to model the special elastomer characteristics requires the use of sophisticated material models and non-linear Finite element analysis tools that are different in scope and theory than those used for metal analysis. Elastomers also call for superior analysis methodologies as elastomers are generally located in a system comprising of metal-elastomer parts giving rise to contact-impact and complex boundary conditions. The presence of these conditions require a judicious use of the available element technology and

solution techniques. The inability to apply a failure theory as applicable to metals increases the complexities regarding the failure and life prediction of an elastomer part. The advanced material models available today define the material as hyperelastic and fully isotropic. The strain energy density (W) function is used to describe the material behavior.

1.2 FEA Support Testing

Most commercial FEA software packages use a curve-fitting procedure to generate the material constants for the selected material model. The input to the curve-fitting procedure is the stress-strain or stress-stretch data from the following physical tests:

- Uniaxial tension test
- Uniaxial compression test
- Planar shear test
- Equibiaxial tension test
- Volumetric compression test

A minimum of one test data is necessary, however greater the amount of test data, better the quality of the material constants and the resulting simulation. Testing should be carried out for the deformation modes the elastomer part may experience during its service life. To ensure a quasi-static process, the physical testing is carried out at a speed of 0.2 inch/minute or 0.084 mm/sec. The material can also be aged in a liquid or at elevated temperatures before testing and thus service conditions can be incorporated to generate the material constants and subsequent FE analysis.

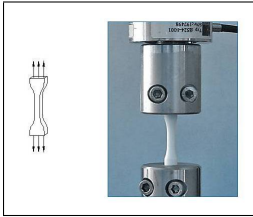


Figure 1.1: Uniaxial Tension Test

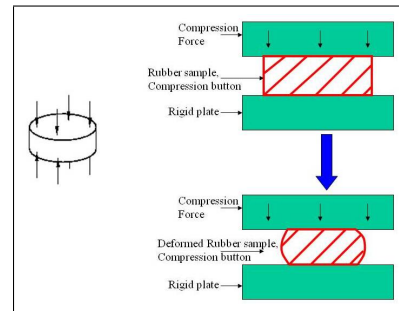


Figure 1.2: Uniaxial Compression Test

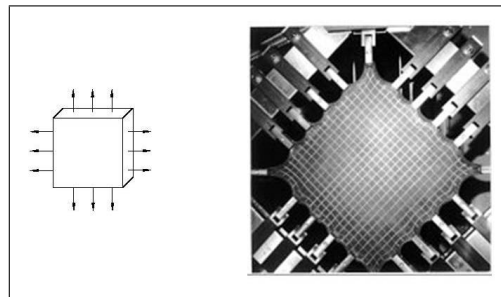


Figure 1.3: Equibiaxial Tension Test

1.2.1 Uniaxial Tension Test

THE TENSION TEST is one of the most commonly used tests for evaluating materials. In its simplest form, the tension test is accomplished by gripping opposite ends of a test item within the load frame of a test machine. A tensile force is applied by the machine, resulting in the gradual elongation and eventual fracture of the test item.

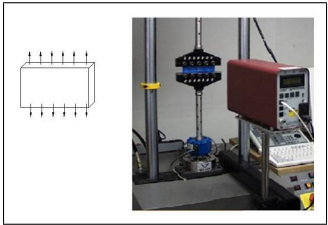


Figure 1.4: Planar Shear Test

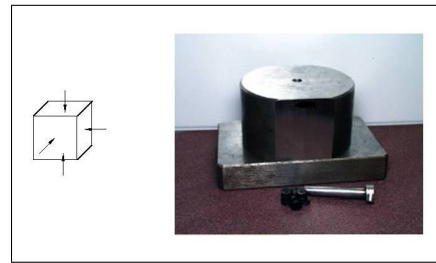


Figure 1.5: Volumetric Compression Test

Table(1.1) shows the equivalent tests that can be carried out to characterize the elastomeric materials. Certain tests that pose a difficulty can be replaced by its equivalent test.

Number	Test	Equivalent Test
1	Uniaxial Tension	Equibiaxial Compression
2	Uniaxial Compression	Equibiaxial Tension
3	Planar Tension	Planar Compression

Table 1.1: Equivalent FEA Support Tests

1.3 Strain Energy Density Functions and Material Models

In the FE analysis of elastomeric materials, the material is characterized by using different forms of the strain energy density function. The strain energy density of a solid can be defined as the work done per unit volume to deform a material from a stress free reference or original state to a final state. The strain energy density functions have been derived using a Statistical mechanics, and Continuum mechanics involving Invariant and Stretch based approaches.

1.3.1 Statistical Mechanics Approach using Gaussian and Non-Gaussian Models

The statistical mechanics approach is based on the assumption that the elastomeric material is made up of randomly oriented molecular chains. In the Gaussian model the total end to end length of a chain (r) is given by

$$P(r) = 4\pi \left(\frac{3}{2\pi nl^2} \right)^{\frac{3}{2}} r^2 \exp \left(-\frac{3r^2}{2nl^2} \right) \quad (1.1)$$

where n is the number of chains in the link and l is the length of each link. The initial length of the chain is given by the root mean square value of r

$$L_0 = \left(\bar{r^2} \right)^{\frac{1}{2}} = (nl^2)^{\frac{1}{2}} = \sqrt{nl} \quad (1.2)$$

When force is applied the chain structure deforms and if the deformation of N chains by a principal stretch state $(\lambda_1, \lambda_2, \lambda_3)$ and the deformation is such that the chain length r is less than its full length nl . The strain energy function W is derived from the change in the entropy in the unloaded (free) and loaded state as

$$W = \frac{1}{2}Nk\theta (\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3) \quad (1.3)$$

where k is the Boltzmann's constant and θ is the absolute temperature. Equation(1.3) is true only in the condition that r is significantly less than the total length nl ($r \ll nl$). At higher deformations of $r/nl \geq 0.40$ the actual force-deflection behavior departs significantly from the one predicted using the Gaussian model. Hence at higher deformations the non-Gaussian nature of the chain stretch has to be taken into account. The resulting non-Gaussian force-extension relationship for a chain is given by

$$f = \frac{k\theta}{l} \mathcal{L}^{-1} \frac{r}{nl} = \frac{k\theta}{l} \mathcal{L}^{-1} \left(\frac{\lambda}{\sqrt{n}} \right) \quad (1.4)$$

where the inverse Langevin function $\mathcal{L}^{-1} \frac{r}{nl}$ is defined as

$$\frac{r}{nl} = \coth \beta - \frac{1}{\beta} = \mathcal{L} \beta; \quad \beta = \mathcal{L}^{-1} \left(\frac{r}{nl} \right) \quad (1.5)$$

To obtain a model that co-relates the extension of the individual chains to the applied force-deformation a network structure is established. Figure(1.6) shows the network models. The difference in all the three models is how the deformation of the chains is co-related to the deformation of the unit cell. In the "3-chain" model the chains are located along the axes of the cubic cell. The chains deform uniformly with the cell and the stretch on each chain then corresponds to the principal stretch value. The strain energy density function is then given by

$$W = \frac{Nk\theta}{3} \sqrt{n} \left[\lambda_1 \beta_1 + \sqrt{n} \ln \left(\frac{\beta_1}{\sinh \beta_1} \right) + \lambda_2 \beta_2 + \sqrt{n} \ln \left(\frac{\beta_2}{\sinh \beta_2} \right) + \lambda_3 \beta_3 + \sqrt{n} \ln \left(\frac{\beta_3}{\sinh \beta_3} \right) \right] \quad (1.6)$$

where $\beta_i = \mathcal{L}^{-1} \left(\frac{\lambda_i}{\sqrt{n}} \right)$; for $i=1,2,3$. In the "8-chain" model also known as the Arruda-Boyce model, the chains are located along the diagonals of the unit cell and deform with

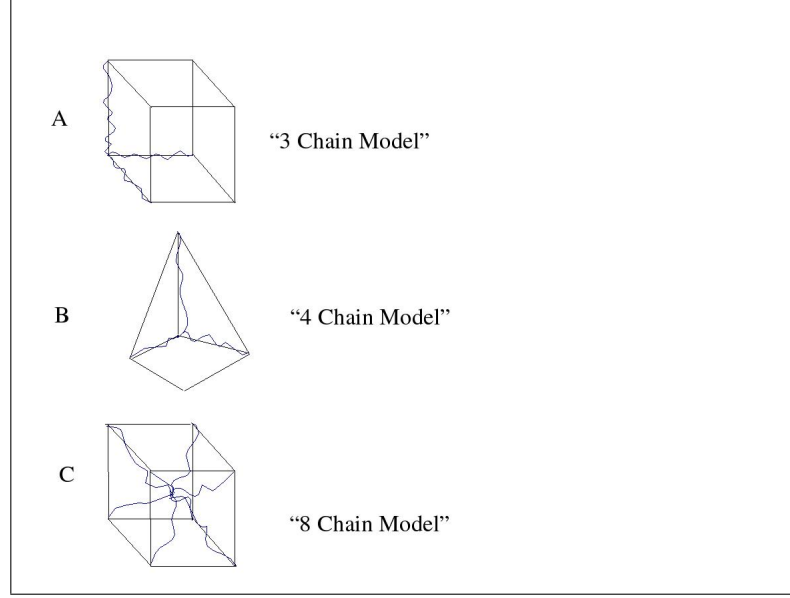


Figure 1.6: 3-chain, 4-chain and 8-chain Network Models

the cell. Due to the symmetry of the chain structure, the interior point remains at the same location through out the applied deformation and the stretch in each individual chain is equal to the root mean-square of the applied stretches and the strain energy function is given by.

$$W = Nk\theta\sqrt{n} \left[\beta_{chain}\lambda_{chain} + \sqrt{n}\ln \left(\frac{\beta_{chain}}{\sinh \beta_{chain}} \right) \right] \quad (1.7)$$

where $\beta_{chain} = \mathcal{L}^{-1} \left(\frac{\lambda_{chain}}{\sqrt{n}} \right)$. It is important to note that the Arruda-Boyce strain energy function is dependent on λ_{chain} which is equal to $\sqrt{I_1/3}$. The strain energy equation shown above can be expanded in polynomial form to obtain

$$W = \mu \sum_{i=1}^5 \frac{C_i}{\lambda_m^{2i-2}} (I_1 - 3) + \frac{1}{D} \left(\frac{J^{el2} - 1}{2} - \ln J^{el} \right) \quad (1.8)$$

where

$$C_1 = \frac{1}{2}; C_2 = \frac{1}{20}; C_3 = \frac{11}{1050}; C_4 = \frac{19}{7000}; C_5 = \frac{519}{673750}$$

μ and λ_m are the material coefficients. The second term in Equation(1.8) depends on the compressibility of the material and J^{el} is the elastic volume ratio of the material.

1.3.2 Invariant Based Continuum Mechanics Approach

The Invariant based continuum mechanics approach is based on the assumption that for a isotropic, hyperelastic material the strain energy density function must depend on the stretch ratio. The three different strain invariants can be defined as

$$\begin{aligned} I_1 &= \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \\ I_2 &= \lambda_1^2 \lambda_2^2 + \lambda_2^2 \lambda_3^2 + \lambda_1^2 \lambda_3^2 \\ I_3 &= \lambda_1^2 \lambda_2^2 \lambda_3^2 \end{aligned} \quad (1.9)$$

where I_1, I_2, I_3 are the three strain invariants. $\lambda_1, \lambda_2, \lambda_3$ are the three principal stretch ratios. The stretch ratio can be defined as

$$\lambda_i = 1 + \epsilon \quad (1.10)$$

where the strain measure generally used is the engineering strain. The use of stretch ratios as a deformation measure in elastomer theory is because of the convenience it provides due to the large magnitudes of deformations taking place. A general form of the strain energy density function can be given as

$$W(I_1, I_2, I_3) = \sum_{ijk=0}^N C_{ijk} (I_1 - 3)^i (I_2 - 3)^j (I_3 - 3)^k + \sum_{i=1}^N \frac{1}{D_i} (J^{el} - 1)^{2i} \quad (1.11)$$

As a result of the assumption about the incompressible nature of the elastomeric material (no volumetric change) the third strain invariant $(I_3) = 1$ and the strain energy function is dependent on only I_1 and I_2 . In Equation(1.11) C_{ijk} and D_i are material parameters, D_i defines the compressibility of the material. For an incompressible material the value of $D_i = 0$. Thus the equation reduces to

$$W(I_1, I_2) = \sum_{ij=0}^N (I_1 - 3)^i (I_2 - 3)^j \quad (1.12)$$

The Neo-Hookean material model, which is one of the simplest material model can be obtained when all C_{ij} with $j \neq 0$ are set to zero

$$W = C_{10}(I_1 - 3) \quad (1.13)$$

with $N = 1$, the Mooney-Rivlin form for the strain energy function is obtained as below

$$W = C_{10}(I_1 - 3) + C_{01}(I_2 - 3) \quad (1.14)$$

The Mooney-Rivlin form can be considered as an improvement over the Neo-Hookean form in that it also depends on the second strain invariant(I_2). It is important to note that both Neo-Hookean and Mooney-Rivlin forms use a linear function of the invariants. The removal of the second strain invariant(I_2) from the Equation(1.11) reduces the function to a reduced polynomial form. The reduced polynomial function with $N = 3$ defines the Yeoh form of the strain energy function as below.

$$W = \sum_{i=1}^3 C_{i0} (I_1 - 3)^i + \sum_{i=1}^3 \frac{1}{D_i} (J^{el} - 1)^{2i} \quad (1.15)$$

1.3.3 Stretch Based Continuum Mechanics Approach

The Stretch based continuum mechanics approach is based on the assumption that the strain energy potential can be expressed as a function of the principal stretches rather than the invariants. The Stretch based Ogden form of the strain energy function is defined as

$$W = \sum_{i=1}^N \frac{2\mu_i}{\alpha_i^2} (\lambda_1^{-\alpha_i} + \lambda_2^{-\alpha_i} + \lambda_3^{-\alpha_i} - 3) + \sum_{i=1}^N \frac{1}{D_i} (J^{el} - 1)^{2i} \quad (1.16)$$

where μ_i and α_i are material parameters and for an incompressible material $D_i = 0$. The choice of the material model depends heavily on the material and the stretch ratios (strains) to which it will be subjected to during its service life. As a thumb rule for small strains of approximately 100 % or $\lambda \leq 2$, simple models as Mooney-Rivlin are sufficient but for higher strains a higher order material model as the Ogden model may be required to successfully simulate the "upturn" or strengthening that can occur in some materials at higher strains. The theory of elastomeric materials and testing methodologies can also be applied to biomaterials including human tissues and polymeric materials used in medical implants (e.g. cardiac pacemaker seals, artificial spinal discs artificial knee joints etc). These materials share some of the complexities like large deformations, viscoelastic behavior with elastomeric materials and the forementioned testing methodologies can be used to characterize the materials for Finite Element Analysis. The study and use of hyperelastic elastomeric materials thus encompasses the state of the art and has wide area of applications ranging from the automotive, aerospace, to modern biomedical applications.

1.4 Curve-Fitting

The stress-strain data from the FEA support tests is used in generating the material constants using a curve-fitting procedure. The constants are obtained by comparing the stress-strain results obtained from the material model to the stress-strain data from experimental tests. Iterative procedure using least-squares fit method is used to obtain the constants, which reduces the relative error between the predicted and experimental values. The linear least squares fit method is used for material models that are linear in their coefficients e.g Neo-Hookean, Mooney-Rivlin, Yeoh etc. For material models that are nonlinear in the coefficient relations e.g. Ogden etc, a nonlinear least squares method is used. For the biaxial test the relations are of the form

$$\begin{aligned}\lambda_1 &= \lambda_2 = \lambda_B, \lambda_3 = \frac{1}{\lambda^2} \\ S_B &= \frac{\partial U}{\partial \lambda_B} \\ S_B &= 2 \left(\lambda^2 - \frac{1}{\lambda^4} \right) (C_{10} + \lambda^2 C_{01})\end{aligned}\tag{1.17}$$

where λ_B and S_B are the stretch and nominal stress. Figure(1.7) shows the stress-stretch

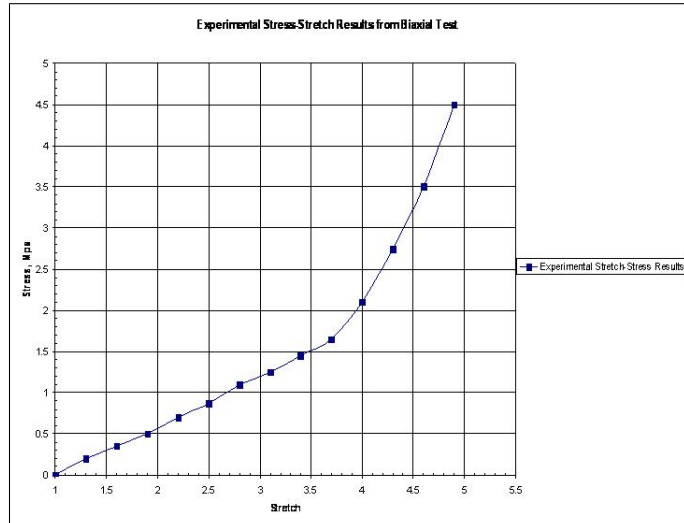


Figure 1.7: Stress Vs. Stretch for Generic Rubber

graph for a generic rubber from a biaxial tension test. To model rubber using Mooney-Rivlin material model it is essential to find the material constants. The method is to assume

a function and calculate the coefficients of the function such that the error in fitting the experimental data points is reduced. The function in the present case would be relation for stress (S_B) shown in Equation(1.17) and the unknowns being the coefficients C_{10} and C_{01} .

1.4.1 Regression Analysis

The goal of a regression analysis is to fit a model to the stress-stretch data. Figure(1.8) shows the results from the regression analysis using least squares procedure for the Mooney-Rivlin constants. Equation(1.17) shows the function that can be used in the case of a biaxial

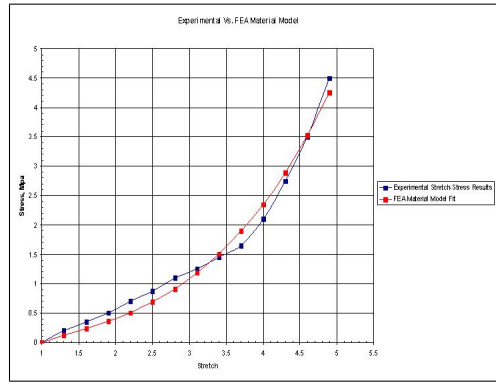


Figure 1.8: Regression Analysis Results for Rubber

test to predict the value of the nominal stress in biaxial state (S_B). Equation(1.17) can be reduced to

$$\sigma = 2 \left(\lambda^2 - \frac{1}{\lambda^4} \right) C_{10} + 2 \left(\lambda^4 - \frac{1}{\lambda^2} \right) C_{01} \quad (1.18)$$

Using linear least-squares procedure to estimate the material parameters C_{10} and C_{01} , the function to be minimized is

$$f = \sum_{i=1}^n (C_{10}\psi_1(\lambda_i) + C_{01}\psi_2(\lambda_i) - \sigma)^2 \quad (1.19)$$

where

$$\begin{aligned} \psi_1(\lambda_i) &= 2 \left(\lambda^2 - \frac{1}{\lambda^4} \right) \\ \psi_2(\lambda_i) &= 2 \left(\lambda^4 - \frac{1}{\lambda^2} \right) \end{aligned} \quad (1.20)$$

The data points that comply with Equation(1.18) must also satisfy the following conditions

$$\frac{\partial F(C_{10}, C_{01})}{\partial C_{10}} = 0 \quad \frac{\partial F(C_{10}, C_{01})}{\partial C_{01}} = 0$$

which is same as

$$\begin{aligned} 2 \sum_{i=1}^n [C_{10}\psi_1(\lambda_i) + C_{01}\psi_2(\lambda_i) - \sigma] \psi_1(\lambda_i) &= 0 \\ 2 \sum_{i=1}^n [C_{10}\psi_1(\lambda_i) + C_{01}\psi_2(\lambda_i) - \sigma] \psi_2(\lambda_i) &= 0 \end{aligned} \quad (1.21)$$

which can be rearranged to obtain

$$\begin{aligned} \underbrace{\left(\sum_{i=1}^n \psi_1^2(\lambda_i) \right)}_a C_{10} + \underbrace{\left(\sum_{i=1}^n \psi_1(\lambda_i) \psi_2(\lambda_i) \right)}_b C_{01} &= \underbrace{\sum_{i=1}^n \psi_1(\lambda_i) \sigma_i}_c \\ \underbrace{\left(\sum_{i=1}^n \psi_1(\lambda_i) \psi_2(\lambda_i) \right)}_d C_{10} + \underbrace{\left(\sum_{i=1}^n \psi_2^2(\lambda_i) \right)}_e C_{01} &= \underbrace{\sum_{i=1}^n \psi_2(\lambda_i) \sigma_i}_f \end{aligned} \quad (1.22)$$

Calculating ψ_1 and ψ_2 using Equation(1.20). The values of the parameters a, b, c, d, e, f

can be found as

$$\begin{aligned} \sum_{i=1}^n \psi_1^2(\lambda_i) &= a = 8710.45 \\ \sum_{i=1}^n \psi_1(\lambda_i) \psi_2(\lambda_i) &= b = 1,58,453.4 \\ \sum_{i=1}^n \psi_1(\lambda_i) \sigma_i &= c = 676.21 \text{ Mpa} \\ \sum_{i=1}^n \psi_1(\lambda_i) \psi_2(\lambda_i) &= d = b = 1,58,453.4 \\ \sum_{i=1}^n \psi_2^2(\lambda_i) &= e = 3,130,107.0 \\ \sum_{i=1}^n \psi_2(\lambda_i) \sigma_i &= f = 12,766.1 \text{ Mpa} \end{aligned} \quad (1.23)$$

The constants for the Mooney-Rivlin material model can now be found out by solving the linear system of equations

$$\begin{bmatrix} 8710.45 & 1,58,453.4 \\ 1,58,453.4 & 3,130,107.0 \end{bmatrix} \begin{Bmatrix} C_{10} \\ C_{01} \end{Bmatrix} = \begin{Bmatrix} 676.21 \\ 12,766.1 \end{Bmatrix} \text{ Mpa} \quad (1.24)$$

which gives

$$C_{10} = 43480.0Pa \quad C_{01} = 1880.0Pa$$

The above method shows a simple way to calculate the material constants analytically. A program in Fortran or any other scientific language can very well be used to calculate the material parameters efficiently.

1.5 Stability of Material Models

1.6 Verification and Validation

In the FEA of elastomeric components it is necessary to carry out checks and verification steps through out the analysis. The verification of the material model and geometry can be carried out in three steps,

- Initially a single element test can be carried out to study the suitability of the chosen material model.
- FE analysis of a tension or compression support test can be carried out to study the material characteristics.
- Based upon the feedback from the first two steps, a verification of the FEA model can be carried out by applying the main deformation mode on the actual component on any suitable testing machine and verifying the results computationally.

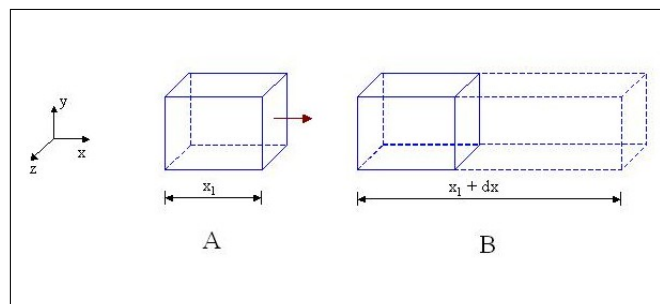


Figure 1.9: Single Element Test

Figure(1.9) shows the single element test for an elastomeric element, a displacement boundary condition is applied on a face, while constraining the movement of the opposite face. Plots A and B show the deformed and undeformed plots for the single element. The load vs. displacement values are then compared to the data obtained from the experimental tests to judge the accuracy of the hyperelastic material model used.

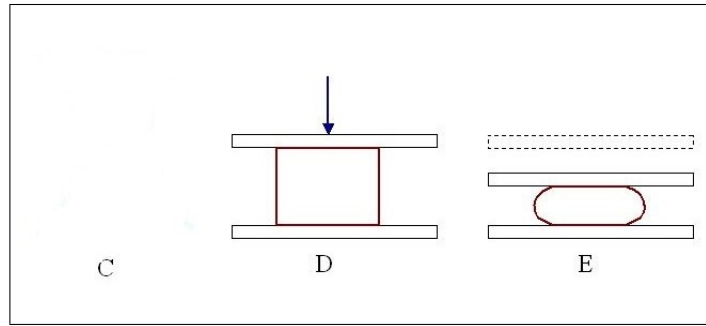


Figure 1.10: Verification Using an FEA Support Test

Figure(1.10) shows the verification procedure carrying out using an FEA support test. Figure shows an axisymmetric model of the compression button. Similar to the single element test, the load-displacement values from the Finite element analysis are compared to the experimental results to check for validity and accuracy. It is possible that the results may match up very well for the single element test but may be off for the FEA support test verification by a margin. Plot C shows the specimen in a testing jig. Plot D and E show the undeformed and deformed shape of the specimen.

Figure(1.11) shows the verification procedure that can be carried out to verify the FEA Model as well as the used material model. The procedure also validates the boundary conditions if the main deformation mode is simulated on an testing machine and results verified computationally. Plot F shows a bushing on a testing jig, plots G and H show the FEA model and load vs. displacement results compared to the experimental results. It is generally observed that verification procedures work very well for plane strain and axisymmetric cases and the use of 3-D modeling in the present procedure provides a more rigorous verification methodology.

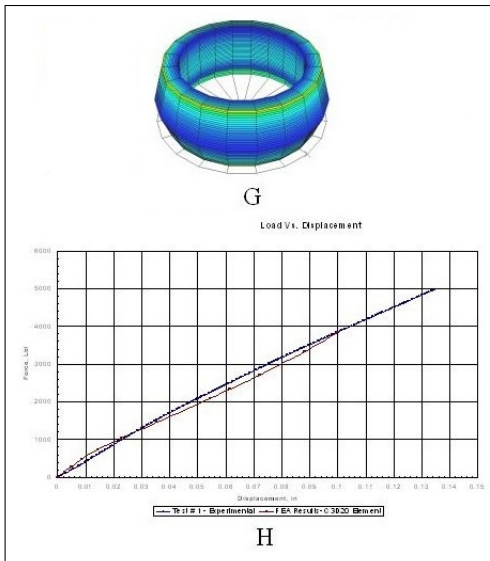


Figure 1.11: FEA Model Verification Using an Actual Part

1.7 Summary

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