

# Verifications and Validations in Finite Element Analysis (FEA)

by  
Kartik Srinivas

Advanced Scientific and Engineering Services (AdvanSES)

(An Independent Material Testing Laboratory)

212, Shukan Mall, Sabarmati-Gandhinagar Highway

Motera, Sabarmati, Ahmedabad 380005

E-mail: [kartik.srinivas@advanses.com](mailto:kartik.srinivas@advanses.com)

Phone: +91-9624447567

<http://www.advanses.com>

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# **Verifications and Validations in Finite Element Analysis (FEA)**

## **1.1 Introduction**

The finite element method (FEM) is a numerical method used to solve a mathematical model of a given structure or system, which are very complex and for which analytical solution techniques are generally not possible, the solution can be found using the finite element method. The finite element method can thus be said to be a variational formulation method using the principle of minimum potential energy where the unknown quantities of interests are approximated by continuous piecewise polynomial functions. These quantities of interest can be different according to the chosen system, as the finite element method can be and is used in various different fields such as structural mechanics, fluid mechanics, acoustics, electromagnetics, etc. In the field of structural mechanics the primary field of interest is the displacements and stresses in the system.

It is important to understand that FEM only gives an approximate solution of the problem and is a numerical approach to get the real result of the variational formulation of partial differential equations. A finite element based numerical approach gives itself to a number of assumptions and uncertainties related to domain discretizations, mathematical shape functions, solution procedures, etc. The widespread use of FEM as a primary tool has led to a product engineering lifecycle where each step from ideation, design development, to product optimization is done virtually and in some cases to the absence of even

prototype testing.

This fully virtual product development and analysis methodology leads to a situation where a misinterpreted approximation or error in applying a load condition may be carried out through out the engineering lifecycle leading to a situation where the errors get cumulative at each stage leading to disastrous results. Errors and uncertainties in the application of finite element method (FEM) can come from the following main sources, 1) Errors that come from the inherent assumptions in the Finite element theory and 2) Errors and uncertainties that get built into the system when the physics we are seeking to model get transferred to the computational model. A common list of these kind of errors and uncertainties are as mentioned below;

- Errors and uncertainties from the solver.
- Level of mesh refinement and the choice of element type.
- Averaging and calculation of stresses and strains from the primary solution variables.
- Uncertainty in recreating the geometrical domain on a computer.
- Approximations in the material properties of the model.
- Approximations and uncertainties in the loading and boundary conditions of the model.
- Errors coming from choosing the right solver types for problems, e.g. Solvers for eigen value problems.

The long list of error sources and uncertainties in the procedure makes it desirable that a framework of rules and criteria are developed by the application of which we can make sure that the finite element method performs within the required parameters of accuracy, reliability and repeatability. These framework of rules serve as verification and validation procedures by which we can consistently gauge the accuracy of our models, and sources of errors and uncertainties be clearly identified and progressively improved to achieve greater accuracy in the solutions. Verifications and Validations are required in each and every

development and problem solving FEA project to provide the confidence that the computational model developed performs within the required parameters. The solutions provided by the model are sufficiently accurate and the model solves the intended problem it was developed for.

Verification procedure includes checking the design, the software code and also investigate if the computational model accurately represents the physical system. Validation is more of a dynamic procedure and determines if the computational simulation agrees with the physical phenomenon, it examines the difference between the numerical simulation and the experimental results. Verification provides information whether the computational model is solved correctly and accurately, while validation provides evidence regarding the extent to which the mathematical model accurately correlates to experimental tests.

In addition to complicated discretization functions, partial differential equations representing physical systems, CFD and FEA both use complicated matrices and PDE solution algorithms to solve physical systems. This makes it imperative to carry out verification and validation activities separately and incrementally during the model building to ensure reliable processes. In order to avoid spurious results and data contamination giving out false signals, it is important that the verification process is carried out before the validation assessment. If the verification process fails the the model building process should be discontinued further until the verification is established. If the verification process succeeds, the validation process can be carried further for comparison with field service and experimental tests.

## **1.2 Brief History of Standards and Guidelines for Verifications and Validations**

Finite element analysis found widespread use with the release of NASA Structural Analysis Code in its various versions and flavours. The early adopters for FEA came from the aerospace and nuclear engineering background. The first guidelines for verification and validation were issued by the American Nuclear Society in 1987 as *Guidelines for the Verification and Validation of Scientific and Engineering Computer Programs for the Nuclear*

*Industry*. The first book on the subject was written by Dr. Patrick Roache in 1998 titled *Verification and Validation in Computational Science and Engineering*, an update of the book appeared in 2009.

In 1998 the Computational Fluid Dynamics Committee on Standards at the American Institute of Aeronautics and Astronautics released the first standards document *Guide for the Verification and Validation of Computational Fluid Dynamics Simulations*. The US Department of Defense through Defense Modeling and Simulation Office released the *DoD Modeling and Simulation, Verification, Validation, and Accreditation Document* in 2003.

The American Society of Mechanical Engineers (ASME) V and V Standards Committee released the *Guide for Verification and Validation in Computational Solid Mechanics (ASME V and V-10-2006)*.

In 2008 the National Aeronautics and Space Administration *Standard for Models and Simulations* for the first time developed a set of guidelines that provided a numerical score for verification and validation efforts.

American Society of Mechanical Engineers V and V Standards Committee V and V-20 in 2016 provided an updated *Standard for Verification and Validation in Computational Fluid Dynamics and Heat Transfer*.

### **1.3 Verifications and Validations :- Process and Procedures**

Figure(1.1) shows a typical product design cycle in a fast-paced industrial product development group. The product interacts with the environment in terms of applied loads, boundary conditions and ambient atmosphere. These factors form the inputs into the computational model building process. The computational model provides us with predictions and solutions of what would happen to the product under different service conditions.

It is important to note that going from the physical world to generating a computational model involves an iterative process where all the assumptions, approximations and their effects on the the quality of the computational model are iterated upon to generate the most optimum computational model for representing the physical world.

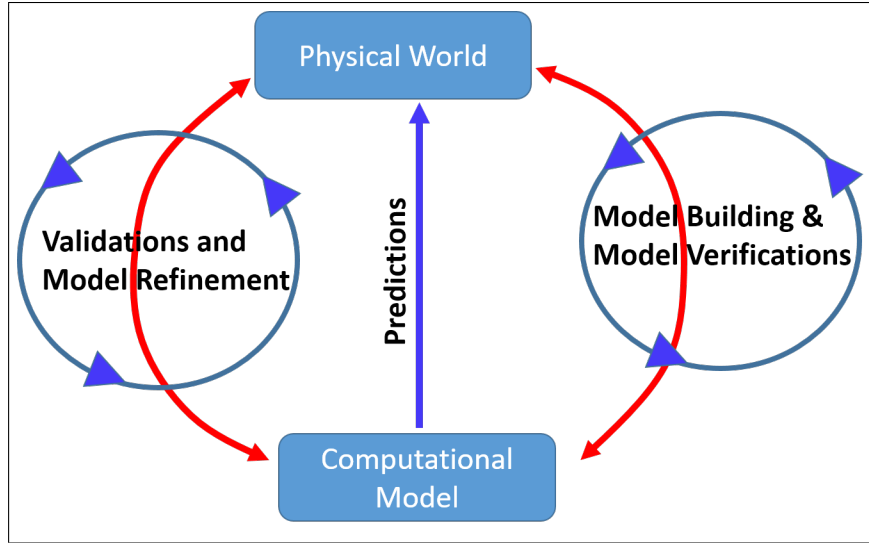


Figure 1.1: Variation on the Sargent Circle Showing the Verification and Validation Procedures in a Typical Fast Paced Design Group

The validation process between the computational model and the physical world also involves an iterative process, where experiments with values of loads and boundary conditions are solved and the solution is compared to output from the physical world. The computational model is refined based upon the feedbacks obtained during the procedure.

The circular shapes of the process representation emphasizes that computational modeling and in particular verification and validation procedures are iterative in nature and require a continual effort to optimize them.

The blue, red and green colored areas in Figure(1.3) highlight the iterative validation and verification activities in the process. The standards and industrial guidelines clearly mention the distinctive nature of code and solution verifications and validations at different levels. The green highlighted region falls in the domain of the laboratory performing the experiments, it is equally important that the testing laboratory understands both the process and procedure of verification and validation perfectly.

Code verification seeks to ensure that there are no programming errors and that the code yields the accuracy expected of the numerical algorithms used to approximate the solutions of the underlying differential equations. This is in contrast with calculation verification, which is concerned with estimating the discretization error in the numerical solution of



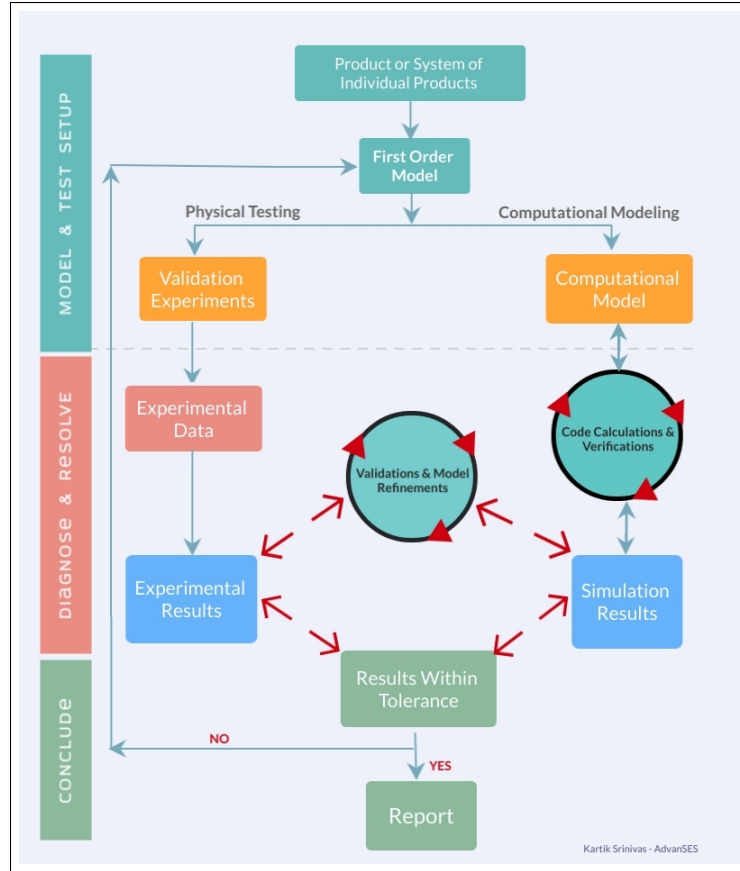


Figure 1.2: Verification and Validation Process

the specific problem of interest. The distinction is subtle but important, because code verification requires an independent, highly accurate reference solution and can (and usually will) operate on a problem that is different from the problem of interest. What is important in code verification is that all portions of the code relevant to the problem at hand be fully checked to ensure that they are mistake free. This is done by comparing numerical results with analytical solutions, and in the process, confirming that the numerical solution converges to the exact one at the expected rate as the mesh is refined.

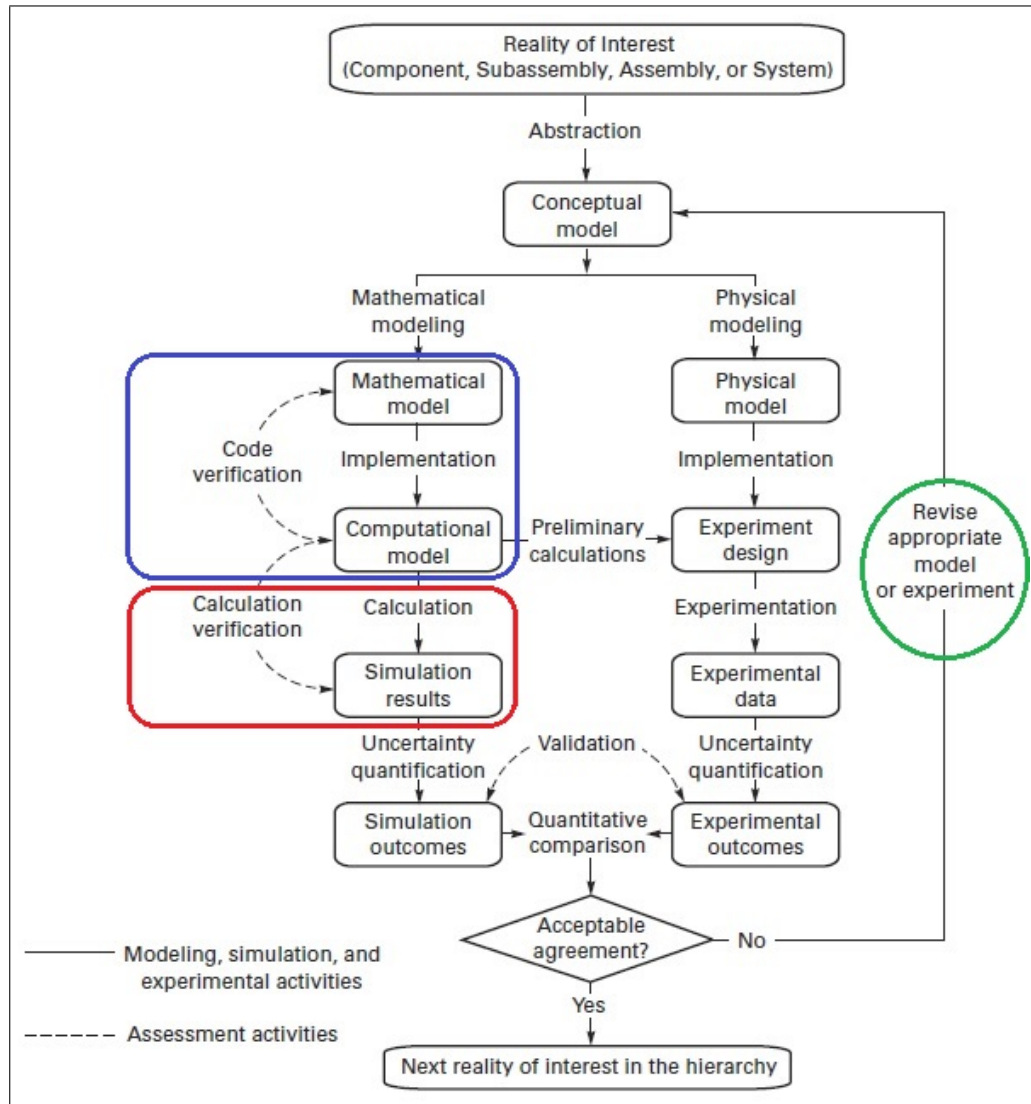


Figure 1.3: Guidance for Verification and Validation as per ASME 10.1 Standard

## 1.4 Guidelines for Verifications and Validations

The first step is the verification of the code or software to confirm that the software is working as it was intended to do. The idea behind code verification is to identify and remove any bugs that might have been generated while implementing the numerical algorithms or because of any programming errors. Code verification is primarily a responsibility of the code developer and softwares like Abaqus, LS-Dyna etc., provide example problems manuals, benchmark manuals to show the verifications of the procedures and algorithms they have implemented.

Next step of calculation verification is carried out to quantify the error in a computer simulation due to factors like mesh discretization, improper convergence criteria, approximation in material properties and model generations. Calculation verification provides with an estimation of the error in the solution because of the mentioned factors. Experience has shown us that insufficient mesh discretization is the primary culprit and largest contributor to errors in calculation verification.

Validation processes for material models, elements, and numerical algorithms are generally part of FEA and CFD software help manuals. However, when it comes to establishing the validity of the computational model that one is seeking to solve, the validation procedure has to be developed by the analyst or the engineering group.

The following validation guidelines were developed at Sandia National Labs [Oberkampf et al.] by experimentalists working on wind tunnel programs, however these are applicable to all problems from computational mechanics.

**Guideline 1:** The validation experiment should be jointly designed by the FEA group and the experimental engineers. The experiments should ideally be designed so that the validation domain falls inside the application domain.

**Guideline 2:** The designed experiment should involve the full physics of the system, including the loading and boundary conditions.

**Guideline 3:** The solutions of the experiments and from the computational model should be totally independent of each other.

**Guideline 4:** The experiments and the validation process should start from the system

level solution to the component level.

**Guideline 5:** Care should be taken that operator bias or process bias does not contaminate the solution or the validation process.

## 1.5 Verification & Validation in FEA

### 1.5.1 Verification Process of an FEA Model

In the case of automotive product development problems, verification of components like silent blocks and bushings, torque rod bushes, spherical bearings etc., can be carried. Figure(1.4) shows the rubber-metal bonded component for which calculations have been carried out. Hill[11], Horton[12] and have shown that under radial loads the stiffness of the bushing can be given by,

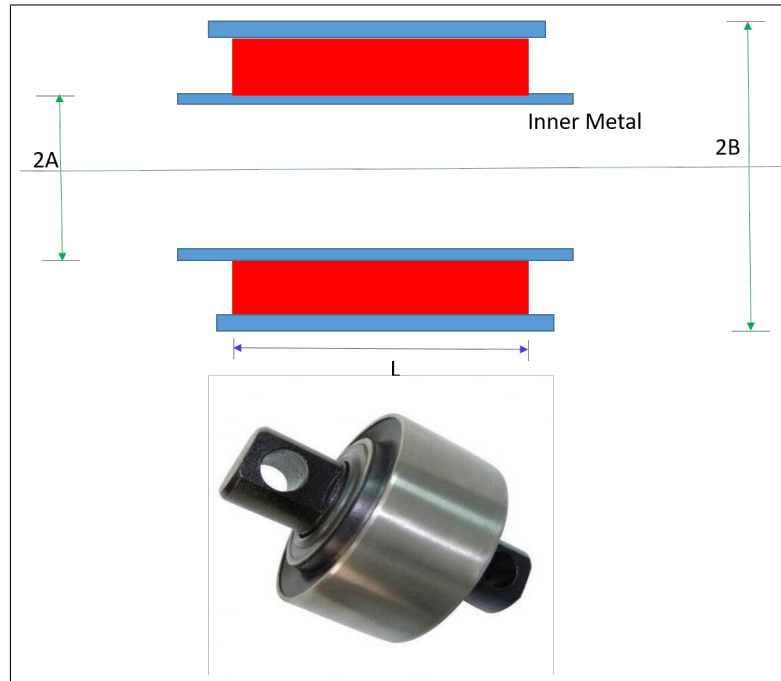


Figure 1.4: Geometry Dimensions of the Silent Bushing

$$K_{rs} = \beta_{rs} LG \quad (1.1)$$

$$\text{where, } \beta_{rs} = \frac{80\pi (A^2 + B^2)}{25 (A^2 + B^2) \ln \left( \frac{B}{A} \right) - 9 (A^2 - B^2)} \quad (1.2)$$

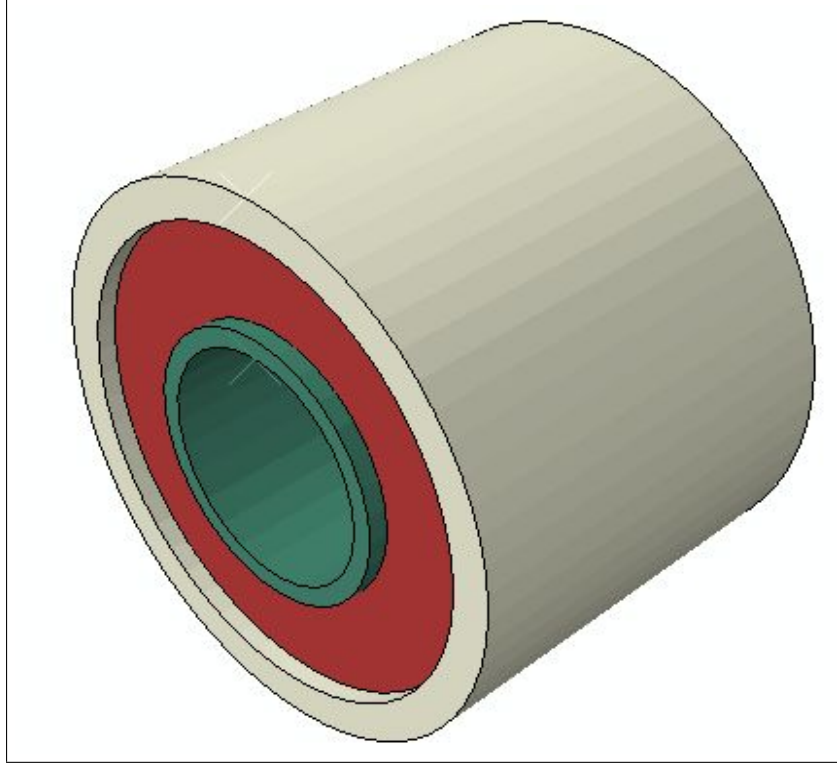


Figure 1.5: Geometry of the Silent Bushing

and  $G = \text{Shear Modulus} = 0.117e^{0.034xH_s}$ ,  $H_s = \text{Hardness of the material}$ . Replacing the geometrical values from Figure(1.4),

$$K_{rs} = 8170.23 \text{ N/mm}, \quad (1.3)$$

for a 55 durometer natural rubber compound. The finite element model for the bushing is shown in Figure(1.9) and the stiffness from the FEA comes to 8844.45 N/mm. The verification and validation quite often recommends that a difference of less than 10% for a comparison of solutions is a sound basis for a converged value.

For FEA with non-linear materials and non-linear geometrical conditions, there are multiple steps that one has to carry out to ensure that the material models and the boundary conditions provide reliable solutions.

- Unit Element Test: The unit element test as shown in Figure(1.7) shows a unit cube element. The material properties are input and output stress-strain plots are compared to the inputs. This provides a first order validation of whether the material properties

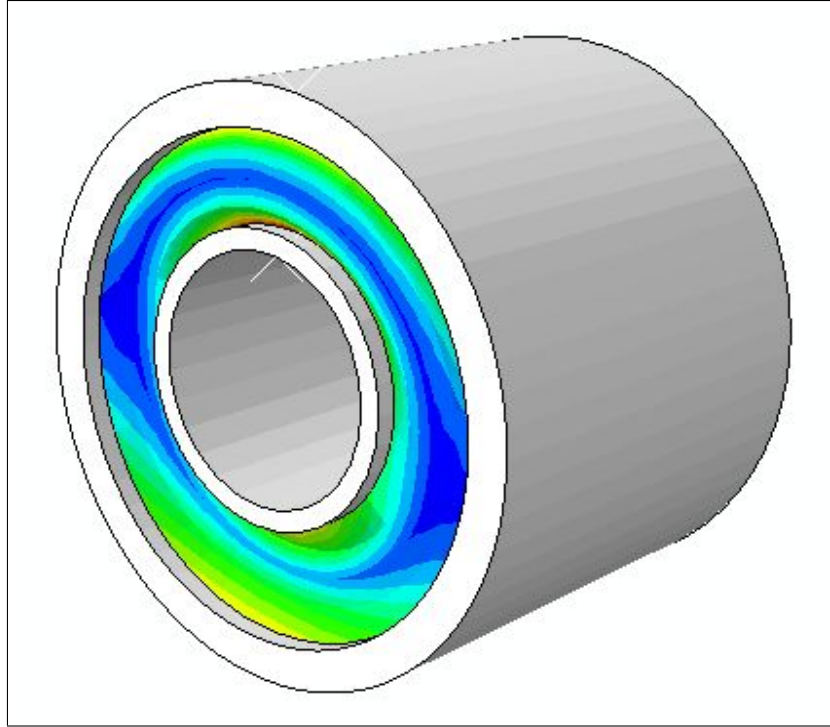


Figure 1.6: Deformed Shape of the Silent Bushing

are good enough to provide sensible outputs. The analyst him/her self can carry out this validation procedure.

- **Experimental Characterization Test:** FEA is now carried out on a characterization test such as a tension test or a compression test. This provides a checkpoint of whether the original input material data can be backed out from the FEA. This is a moderately difficult test as shown in Figure(1.8). The reasons for the difficulties are because of unquantified properties like friction and non-exact boundary conditions.
- **Comparison to Full Scale Experiments:** In these validation steps, the parts and component products are loaded up on a testing rig and service loads and boundary conditions are applied. The FEA results are compared to these experiments. This step provides the most robust validation results as the procedure validates the finite element model as well as the loading state and boundary conditions. Figure(1.9) shows torque rod bushing and the validation procedure carried out in a multi-step analysis.

Experience shows that it is best to go linearly in the validation procedure from step 1

through 3, as it progressively refines one's material model, loading, boundary conditions. Directly jumping to step 3 to complete the validation process faster adds upto more time with errors remaining unresolved, and these errors go on to have a cumulative effect on the quality of the solutions.

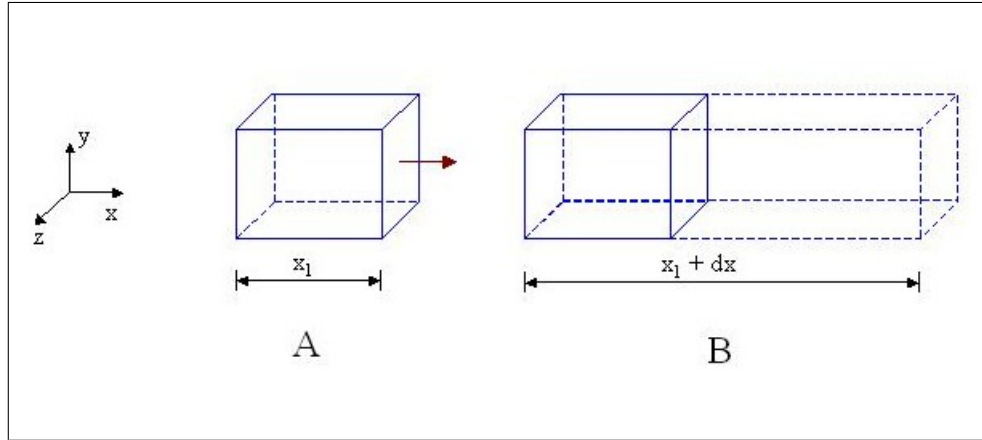


Figure 1.7: Unit Cube Single Element Test

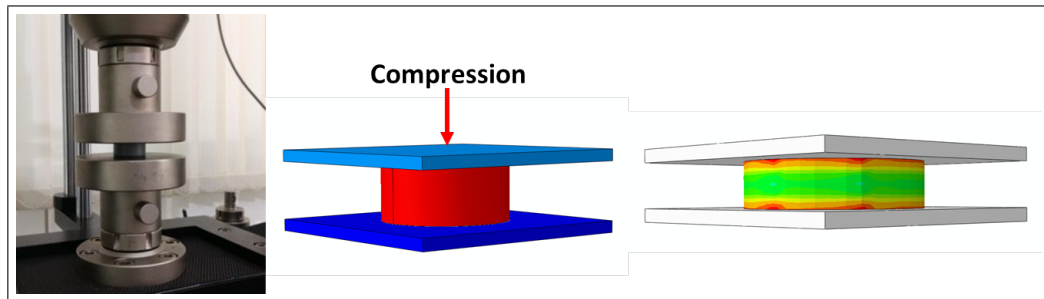


Figure 1.8: FEA of Compression Test

### 1.5.2 Validation Process of an FEA Model

Figure(1.7) shows the experimental test setup for the validation of the bushing model. The radial loading is chosen to be the deformation mode and load vs. displacement results are compared. The verification process earlier carried out established the veracity of the FEA model and the current validation analysis applies the loading in multiple KNs. Results show a close match between the experimental and FEA results. Figures(1.10) and (1.11)

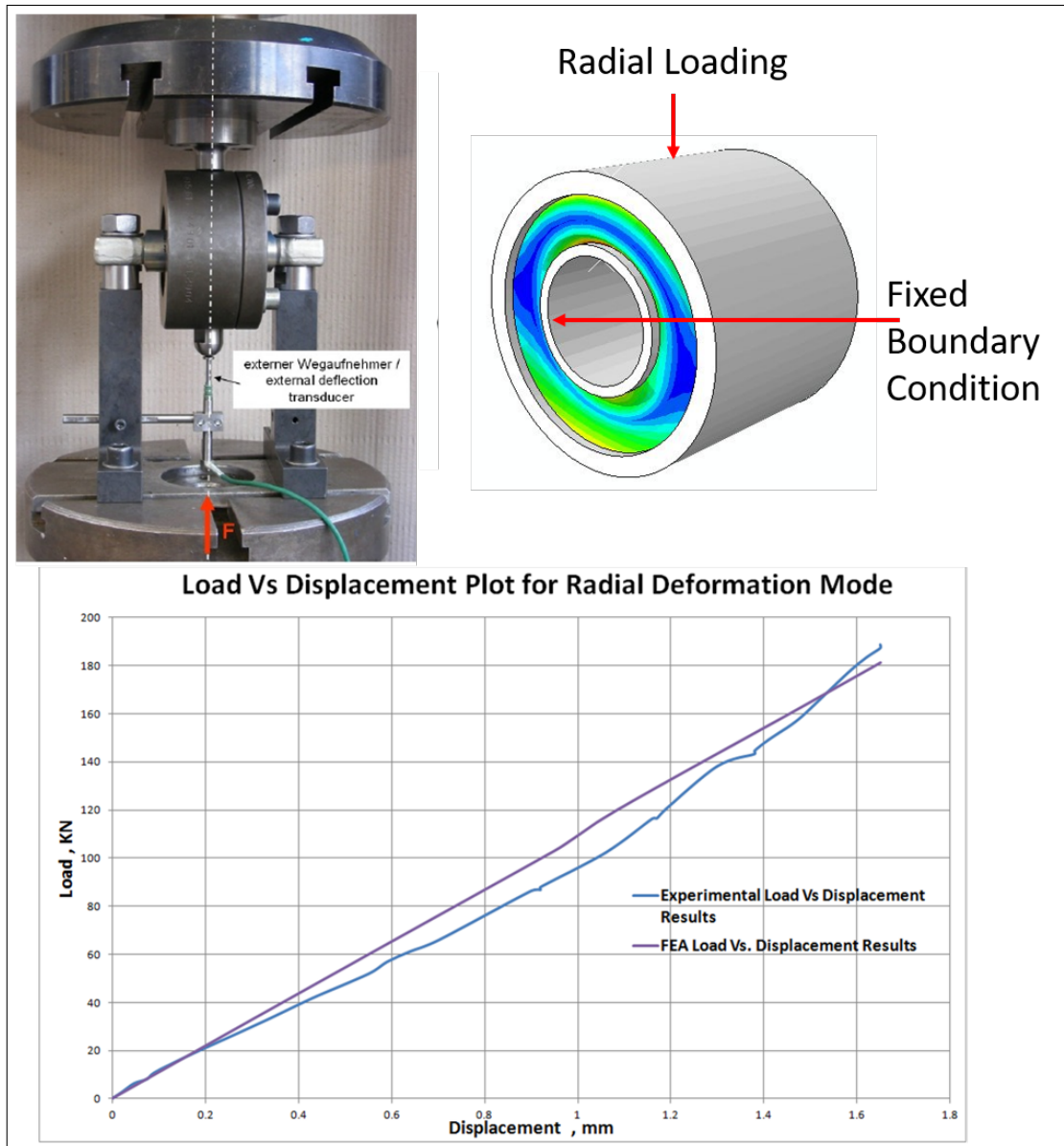


Figure 1.9: Experimental Testing and Validation FEA for the Silent Bushing

show the validation setup and solutions for a tire model and engine mount. The complexity of a tire simulation is due to the nature of the tire geometry, and the presence of multiple rubber compounds, fabric and steel belts. This makes it imperative to establish the validity of the simulations.



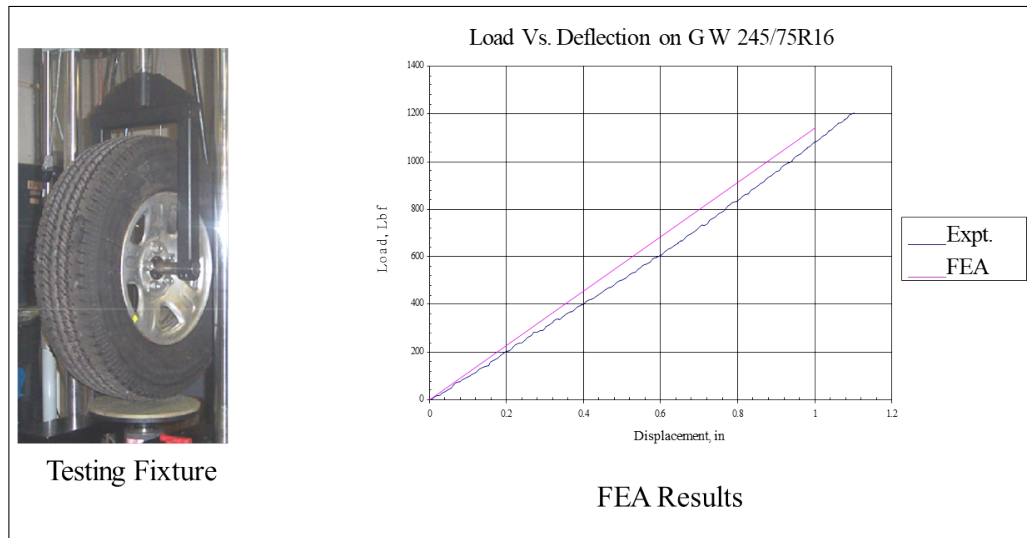


Figure 1.10: Experimental Testing and Validation FEA for a Tire Model

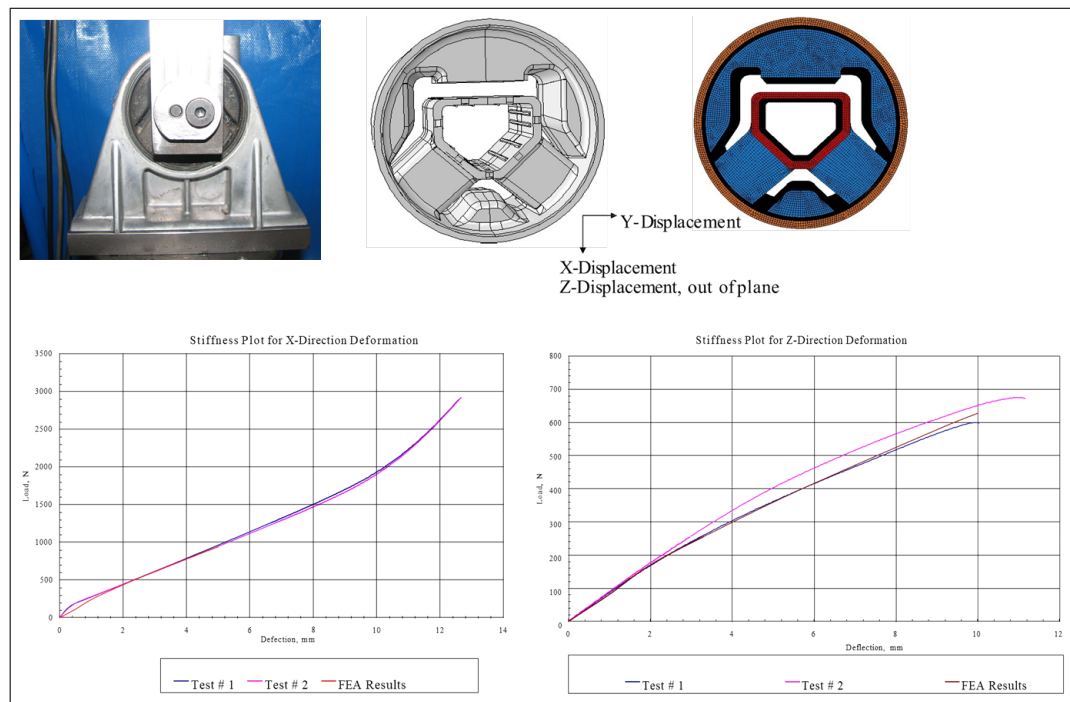


Figure 1.11: Experimental Testing and Validation FEA for a Passenger Car Engine Mount

## 1.6 Summary

An attempt was made in the article to provide information on the verification and validation processes in computational solid mechanics. We went through the history of adoption of verification and validation processes and their integration in computational mechanics processes and tools. Starting from 1987 when the first guidelines were issued in a specific field of application, today we are at a stage where the processes have been standardized and all major industries have found their path of adoption.

Verification and validations are now an integral part of computational mechanics processes to increase integrity and reliability of the solutions. Verification is done primarily at the software level and is aimed at evaluating whether the code has the capability to offer the correct solution to the problem, while validation establishes the accuracy of the solution. ASME, Nuclear Society and NAFEMS are trying to make the process more standardized, and purpose driven.

Uncertainty quantification has not included in this current review, the next update of this article will include steps for uncertainty quantification in the analysis.

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